Biomedical Admissions Test (BMAT)

Section 2: Mathematics
Questions by Topic

M2: Number

M2: Number - Questions by Topic

Mark scheme and explanations at the end

1 If x>10000 then what is the closest value of $\frac{x}{(4x+1)}$?

A ∞

B $\frac{1}{4}$

C $\frac{1}{2}$

D $\frac{5000}{10{,}001}$

E $\frac{5}{3}$

2 If $(A-5)(B-4)=0$, which of the following must be true?

1) If $A = 5,\ then\ B \neq 4$
2) $A = 5,\ \ B = 4$
3) If $A \neq 5,\ then\ B = 4$

A 1 only
B 2 only
C 3 only
D None of the above
E All of the above

3 If $x = 0.04$ and $n = 10000$, calculate the value of the following equation:

$$\sqrt{\frac{x(0.2-x)}{n}}$$

A 0.8

B 0.08

C 0.008

D 0.0008

E 0.00008

4 The mean time for this year's Birmingham annual snail race for a group of 15 snails was 25 minutes. The time for a second group of snails were added and the value of the mean went up to 36 minutes.

Which formula represents the relationship between the number of snails in the second group, S, and the mean time, in minutes, of the second group, T?

A $S = \frac{11}{T-15}$

B $S = \frac{11}{T-36}$

C $S = \frac{165}{T-36}$

D $S = \frac{165T}{T-15}$

E $S = \frac{165}{T-15}$

5 Calculate:

$$\frac{3.304 \times 10^{6} + 3.304 \times 10^{3}}{1.652 \times 10^{10}}$$

A 0.000202

B 0.0000202

C 0.000002

D 0.0004

E 0.00004

6 Calculate the product of 187432 with 0.008212.

A 15391.0

B 1539.19

C 153.919

D 15.3919

E 1.53919

7 Chocolate sweets are sold in packets of 40. Mint sweets are sold in packets of 56. Jimmy wants to buy the same number of chocolate sweets as mint sweets.

What is the smallest number of packets of each of the sweets he could buy?

A 14 chocolate and 10 mint
B 7 mint and 5 chocolate
C 7 chocolate and 5 mint
D 6 chocolate and 4 mint
E 10 mint and 14 chocolate

8 Evaluate $\frac{0.8562}{53.857+6.143}$

Give your answer to 3 significant figures.

A 0.0142
B 0.0143
C 0.01427
D 0.01
E 0.01422

9 Find the fraction exactly halfway between $\frac{3}{4}$ and $\frac{5}{6}$.

Give your answer in its simplest form.

A $\frac{5}{16}$
B $\frac{19}{24}$
C $\frac{19}{12}$
D $\frac{15}{48}$
E $\frac{4}{5}$

10 Find the smallest positive integer, k, such that $180k$ is a perfect square.

A 20
B 5
C 10
D 9
E 4

11 Evaluate:

$$0.75mm - 8.8\mu m$$

Give your answer in standard form and in metres.

A 6.62×10^{-4}
B 7.412×10^{-3}
C 7.412×10^{-4}
D 0.0007412
E 0.000662

12 Given that $2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$, evaluate 10^x.

Find the value of 10^x.

A 2.5
B 1.25
C 0.1
D 0.4
E 0.8

13 B varies with the cube of A. When $B = 12,\ A = 4$.
Find the value of A when $B = \frac{81}{16}$

A 3
B $\frac{27}{16}$
C $\frac{64}{16}$
D $\frac{3}{16}$
E 27

14 I have one brother and one sister. If I square root my brother's age, I get my sister's age. In three years, my sister will be half my brother's age.

How old are my brother and sister respectively?

A 3 and 9
B 5 and 25
C 4 and 16
D 16 and 4
E 9 and 3

15 If $x.y = (x^2y) + (x - y)$, calculate the value of $(2.3).4$

A 19.46
B 44
C 121.46
D 334
E 491

16 Evaluate $\frac{-((-4)^2 - 3\times4)^2}{(-3)^3 + 2\times8}$

A $\frac{16}{11}$
B $\frac{784}{11}$
C $-\frac{16}{11}$
D $\frac{4}{43}$
E $-\frac{784}{11}$

17 A rectangular block has a square base. The length of each side of the base is $(\sqrt{3} - \sqrt{2})$ m and the volume of the block is $(4\sqrt{2} - 3\sqrt{3})\,m^3$.

Find the height of the block in the form $(a\sqrt{3} - b\sqrt{2})\,m$, where a and b are integers.

Find the value of a and of b.

A $a = -1,\ b = -2$
B $a = 2,\ b = -1$
C $a = 1,\ b = 2$
D $a = 2,\ b = 1$
E $a = 1,\ b = -2$

18 Consider the equation $\sqrt{a+b\sqrt{3}} = \frac{13}{4+\sqrt{3}}$, where a and b are integers.

Find the value of a and of b.

A $a = -8,\ b = 19$
B $a = 19,\ b = -8$
C $a = -26,\ b = 9$
D $a = 9,\ b = 26$
E $a = 11,\ b = -1$

19 The solution of the equation $x\sqrt{24} = x\sqrt{3} + \sqrt{6}$ can be represented in the form $\frac{a+\sqrt{b}}{7}$.

Find the value of a and of b.

A $a = -8,\ b = -2$
B $a = 4,\ b = 2$
C $a = 12,\ b = 2$
D $a = 144,\ b = 18$
E $a = 12,\ b = 3$

20 Solve the following simultaneous equations

$$9^x(27)^y = 1$$

$$8^y \div (\sqrt{2})^x = 16\sqrt{2}$$

Find the values of x and y

A $x = 1.8,\ y = -1.2$
B $x = -1.8,\ y = 1.2$
C $x = -2,\ y = -1.33$
D $x = 2,\ y = 1.33$
E $x = 0.6,\ y = 1.6$

21 Sarah is older than her brother Edward. Their ages in years are such that twice the square of Edward's age subtracted from the square of Sarah's age gives a number equal to 6 times the difference of their ages. The sum of their ages is equal to 5 times the difference of their ages.

Find the age in years of each of the siblings.

A Sarah is 24 years old, Edward is 18 years old.
B Sarah is 6 years old, Edward is 9 years old.
C Sarah is 18 years old, Edward is 12 years old.
D Sarah is 3 years old, Edward is 6 years old.
E Sarah is 12 years old, Edward is 18 years old.

Solutions

1 **B is the answer**

Divide the numerator and denominator of the fraction by x. (Remember, as long as you are dividing both top and bottom by the same thing, you are not changing the value of the fraction, simply rewriting it in a different form).

$$\frac{x \div x}{(4x+1) \div x} \Rightarrow \frac{1}{4+\frac{1}{x}}$$

With the fraction rewritten like this, we can see clearly that as x becomes larger, i.e. $> 10,000$, the $\frac{1}{x}$ becomes very small and approaches 0, just leaving $\frac{1}{4}$.

2 **C is the answer**

A is incorrect because if $A = 5$ then B can be any number *including* 4 because the product will still equal 0 due to the first bracket $(5-5)$ still equaling 0.

B is incorrect because only one of the brackets has to equal 0 in order for the product of both brackets to still equal 0, therefore if $A = 5$, B can equal any value and if $B = 4$, A can equal any value and the equation will still hold true. $A = 5$ *and* $B = 4$ isn't necessary at the same time for the equation to hold true.

C is correct because if A is not equal to 5, then B MUST equal 4 for the equation to hold true.

D and E are incorrect

3 **D is the answer**

Method 1: Substitute the values for x and n into the given equation

$$\sqrt{\frac{x(0.2-x)}{n}} \Rightarrow \sqrt{\frac{0.04(0.2-0.04)}{10000}} \Rightarrow \sqrt{\frac{0.04(0.16)}{10000}} \Rightarrow \sqrt{\frac{0.0064}{10000}} = \frac{\sqrt{0.0064}}{\sqrt{10000}}$$

To calculate $\sqrt{10000}$:

$$\sqrt{10000} \equiv \sqrt{100 \times 100} = 10 \times 10 = 100$$

To calculate $\sqrt{0.0064}$, multiply $\sqrt{0.0064}$ by $\sqrt{10000}$ to get $\sqrt{64} = 8$. But remember, $\sqrt{0.0064}$ was multiplied by $\sqrt{10000}$ therefore you must UNDO this action and divide 8 by $\sqrt{10000}$ $(= 100)$ to get 0.08

The equation is now $\frac{0.08}{100} = 0.0008$

Method 2: Substitute the values for x and n into the given equation

$$\sqrt{\frac{x(0.2-x)}{n}} \Rightarrow \sqrt{\frac{0.04(0.2-0.04)}{10000}} \Rightarrow \sqrt{\frac{0.04(0.16)}{10000}}$$

Eliminate the decimals in the numerator by multiplying **both** the numerator and denominator by 1000. Then, continue to solve the equation.

$$\sqrt{\frac{0.04(0.16)}{10000}} = \sqrt{\frac{0.04(0.16)\times 10000}{10000\times 10000}} = \sqrt{\frac{4(16)}{100000000}} = \sqrt{\frac{64}{100000000}} = \frac{8}{10000} = 0.0008$$

4 **C is the answer**

$$Total\ mean\ of\ both\ groups = \frac{(Total\ time\ Group\ 1 + Total\ time\ Group\ 2)}{(No.\ of\ snails\ Group\ 1 + No.of\ snails\ Group\ 2)}$$

Total time of snails in Group 1:

$$Mean = \frac{Total\ time}{No.\ of\ snails} \Rightarrow 25 = \frac{Total\ time}{15} \Rightarrow Total\ time = 25 \times 15$$

$$Total\ time\ of\ snails\ in\ Group\ 1 = 375$$

Total time of snails in Group 2:

$$Mean = \frac{Total\ time}{No.\ of\ snails} \Rightarrow T = \frac{Total\ time}{S}$$

(where T = mean time of second group, S = no. of snails in second group)

$$Total\ time\ of\ snails\ in\ Group\ 2 = TS$$

Total mean time of both groups = 36

$$\Rightarrow 36 = \frac{375+TS}{15+S} \Rightarrow 36(15+S) = 375 + TS \Rightarrow 540 + 36S = 375 + TS$$

$$\Rightarrow 165 = TS - 36S \Rightarrow 165 = S(T-36) \Rightarrow S = \frac{165}{T-36}$$

5 **A is the answer**

Notice that 1.652 x 2= 3.304, so the fraction can be cancelled down using **1.652 as a common factor**.

Using 1.652 as a common factor, the fraction can be cancelled down to give:

$$\frac{2\text{x } 10^{6} + 2 \text{ x}10^{3}}{1\text{x } 10^{10}}$$

To complete the division by 1 x 10^{10}, subtract the powers:
The fraction is equivalent to

$$2\text{x}10^{6-10} + 2\text{x}10^{3-10}$$

which becomes

$$2 \text{ x}10^{-4} + 2\text{x}10^{-7}$$
$$= 0.0002 + 0.0000002$$
$$= \mathbf{0.0002002}$$

> ***Exam Tip*** - Try to **look out for tricks like the one in this question** when doing questions involving calculations; there are often **links between different numbers** in a question. For example, one number might be a factor of the other number. Spotting things like this will make the calculation much easier to solve, and will save you valuable time!

6 **B is the answer**

The trick with this question is to notice you are **not required** to calculate the actual numerical values, only the magnitude of the number. If you pick up on this, it can save you a lot of time.

187,432 can be rounded to 200,000 and 0.008212 can be rounded to 0.01

$$\Rightarrow \quad 200{,}000 \times 0.01 = 2000$$

This tells you the magnitude of the number, i.e. there will be 4 digits before the decimal point.

> ***Exam Tip*** - Again, **look out for time-saving tricks** like the one in this question!

7 **C is the answer**

To find the same number of chocolate and mint sweets, find the lowest common multiple (LCM) of 40 and 56.

Method 1:
List the multiples of 40 and 56:

40 x 1 = 40	56 x 1 = 56
40 x 2 = 80	56 x 2 = 112
40 x 3 = 120	56 x 3 = 168
40 x 4 = 160	56 x 4 = 224
40 x 5 = 200	56 x **5** = 280
40 x 6 = 240	
40 x **7** = 280	

Hence, the smallest number of bags of chocolate sweets is 7 and the smallest number of bags of mint sweets is 5.

Method 2:
Find the prime factors of both 40 and 56. This can be done by dividing the value as much as possible and collecting the prime factors on the left hand side of the column:

2	4 0			2	5 6
2	2 0			2	2 8
2	1 0			2	1 4
5	5			7	7
	1				1

$$40 = 2^3 \times 5$$
$$56 = 2^3 \times 7$$

LCM is $2^3 \times 5 \times 7 = 280$

Hence, the smallest number of bags of chocolate sweets is 7 and the smallest number of bags of mint sweets is 5.

Method 3:

This method should be used when comparing **large** numbers, or when the question requires comparison of **3 or more** numbers. This is because it is quite tedious and time-consuming, and you can use the other two methods when comparing two smaller numbers.

Compare the last digits of the numbers involved (4**0** and 5**6**).
The last digit for multiples of 4**0** can only be **0**.

Find the smallest number that 56 can be multiplied by to get a number with the last digit **0**. This is a matter of times table as 6 x 5 = 30. So the number is **5**.
Hence the LCM of 40 and 56 = 56 x 5 = 280.

Work backwards to find the number of bags of chocolate.

$$280 \div 40 = 7$$

Hence, the smallest number of bags of chocolate sweets is 7 and the smallest number of bags of mint sweets is 5.

A is incorrect because the number of bags of sweets are **not** the smallest.
B is incorrect because the number of bags of chocolate sweets have been mixed up with the number of bags of mint sweets.
D is incorrect because the first multiple of 40 and 56 was not written down and resulted in a counting error. Remember that **40 is the first multiple of 40** and **56 is the first multiple of 56**!
E is incorrect because the number of bags of sweets are **not** the smallest and also, the number of bags of chocolate sweets have been mixed up with the number of bags of mint sweets.

8 **B is the answer**

As the answer options are quite similar, you cannot use rounding when working out the answer. Instead, work out the answer step by step. The question involves decimals as numerators and denominators so **removing the decimal points** will allow us to manipulate the values more easily.

Method 1:

$$\frac{0.8562}{53.857+6.143}$$

$$= \frac{0.8562}{60}$$

$$= \frac{0.8562}{60} \times \frac{10000}{10000}$$

$$= \frac{8562}{600000}$$

$$= 0.01427 = 0.0143 \ (3\ s.f.)$$

				0.	0	1	4	2	7
6 0 0 0 0 0	8	5	6	2	0	0			
	6	0	0	0	0	0			
	2	5	6	2	0	0	0		
	2	4	0	0	0	0	0		
		1	6	2	0	0	0	0	
		1	2	0	0	0	0	0	
			4	2	0	0	0	0	0
			4	2	0	0	0	0	0

Method 2:

$$\frac{0.8562}{53.857+6.143}$$

$$= \frac{0.8562}{60}$$
$$= \frac{0.8562}{60} \times \frac{10000}{10000}$$
$$= \frac{8562}{600000}$$
$$= \frac{4281}{300000}$$
$$= \frac{1427}{100000}$$ = 0.01427 = **0.0143** (3 s.f.)

A is incorrect because 0.01427 was **rounded incorrectly**.
C is incorrect because the answer is **not given to 3 s.f.**
D is incorrect because if the 0s appear before a natural number, they do not count as significant figures.
E is incorrect because not all digits immediately following the decimal point count as significant figures.

> ***Exam Tip*** - It is a good practice to **simplify fractions** before manipulating them, especially when dealing with **large** numerators or denominators.
>
> For example, by simplifying $\frac{8562}{600000}$ to $\frac{1427}{100000}$ it makes the division calculations easier and enables you to reach the final answer quicker.

9 **B is the answer**

$$\left(\frac{3}{4} + \frac{5}{6}\right) \div 2$$
$$= \left(\frac{9}{12} + \frac{10}{12}\right) \div 2$$
$$= \left(\frac{19}{12}\right) \times \frac{1}{2}$$
$$= \frac{19}{24}$$

The same answer can also be achieved by using a different common denominator. Another way of getting a common denominator is to **multiply** the two denominators together: $4 \times 6 = 24$.

$$\left(\frac{3}{4} + \frac{5}{6}\right) \div 2 = \left(\frac{18}{24} + \frac{20}{24}\right) \div 2 = \left(\frac{38}{24}\right) \times \frac{1}{2} = \frac{19}{24}$$

C is incorrect - this answer would be obtained by forgetting to divide the sum by 2.

> ***Exam Tip*** - The common denominator does not have to be the LCM of the denominators, any common multiple of the denominators will do.

When dealing with **small** numbers, **multiplying the denominators** together is a convenient way to get a common denominator.

When dealing with **large** numbers, using the **LCM as the common denominator** will make further calculations simpler since you will be dealing with smaller numbers.

10 **B is the answer**

First, express 180 as a product of its prime factors:

Divide 180 by the **smallest** possible prime number

$$180 \div \underline{\mathbf{2}} = 90$$

Then, divide the result (90) by the smallest possible prime number and keep repeating this step until the final result is 1.

$$90 \div \underline{\mathbf{2}} = 45$$
$$45 \div \underline{\mathbf{3}} = 15$$
$$15 \div \underline{\mathbf{3}} = 5$$
$$5 \div \underline{\mathbf{5}} = 1$$

Therefore, 180 = $2^2 \times 3^2 \times 5$

A number that is a **perfect square** is a **product of squares**.

$$180 = 2^2 \times 3^2 \times 5$$

Since only 5 is left unsquared, in order to make 180k a perfect square, multiply 180 by 5 to get a product of squares:

$$180 \times 5 = 2^2 \times 3^2 \times 5^2$$
$$k = 5$$

A is incorrect because it is not the smallest positive integer.
C is incorrect because the product is not a square.
D is incorrect because the product is not a square.
E is incorrect because the product is not a square.

11 **C is the answer**

$$0.75\ mm\ - 8.8\mu m$$
$$= 0.75 \times 10^{-3} - 8.8 \times 10^{-6} m$$
$$= 0.75 \times 10^{3} \times 10^{6} - 8.8 \times 10^{-6} m$$
$$= 750 \times 10^{-6} - 8.8 \times 10^{-6} m$$
$$= 741.2 \times 10^{-6} m$$
$$= 7.412 \times 10^{2} \times 10^{-6} m$$
$$= 7412 \times 10^{-4} m$$

A is incorrect because this answer would be obtained by converting micrometres into 10^{-5} m instead.
B is incorrect because the answer is incorrect by one power of 10.
D is incorrect because it is not in standard form.
E is incorrect because it is not in standard form.

12 **E is the answer**

$$2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$$
$$2^{2x} \cdot 2^2 \times \frac{5^x}{5} = 2^{3x} \times 5^{2x}$$
$$\frac{2^2}{5} = \frac{2^{3x}}{2^{2x}} \times \frac{5^{2x}}{5^x}$$

$$0.8 = 2^x \times 5^x = 10^x$$

A is incorrect.
B is incorrect.
C is incorrect.
D is incorrect.

13 **A is the answer**

The key to solving this question is to be able to identify that "B varies with the cube of A" is another way of saying "**B is directly proportional to the cube of A**". This then becomes a **proportion** question.

Convert the sentence given in the question into a statement of proportionality:

$$B\ \alpha\ A^3$$

Replace α with **'=k'** to form an equation. **'k' represents an unknown constant.** Since this question refers to **direct proportion, the generic equation is** $y = kx$. If this question was about **inverse proportion the generic equation would be** $y = \frac{k}{x}$

$$B = kA^3$$

Use the given values of B and A $(12 \ and \ 4)$ to find k:

$$12 = k4^3 \quad \Rightarrow 12 = 64k \quad \Rightarrow k = \frac{12}{64} \quad \Rightarrow k = \frac{3}{16}$$

Put the value of k back into the equation: $B = \frac{3}{16}A^3$

Use the equation to find the value of A when $B = \frac{81}{16}$

$$\frac{81}{16} = \frac{3}{16}A^3 \quad \Rightarrow 27 = A^3 \quad \Rightarrow A = 3$$

> ***Exam Tip*** - The phrase "varies with" is a phrase you need to know refers to direct proportionality and the equation it corresponds to.

14 **A is the answer**

Set up two **simultaneous equations** from the information provided. Let B stand for brother and S stand for sister.

"If I square root my brother's age, I get my sisters age" $\Rightarrow \sqrt{B} = S$ *(1)*

"In three years, my sister will be half my brother's age" $\Rightarrow S + 3 = \frac{B+3}{2}$ *(2)*

Since we are dealing with algebraic terms and fractions and not just whole numbers, to make the equations easier to work with, square both sides of equation 1 to **eliminate the square root** function.

$$B = S^2 \quad (3)$$

Once you have set up the two simultaneous equations, there are several different methods to solve this. In this method, we will **solve by substitution**.

Rearrange *(2)* to make B the subject:

$2(S+3) = B+3 \quad \Rightarrow \quad 2S+6 = B+3 \quad \Rightarrow \quad B = 2s+3$ *(4)*

Substitute equation *(4)* into *(3):*

$$2s+3 = S^2$$

Solve the quadratic equation to find a value for S:

$S^2 - 2s - 3 = 0 \quad \Rightarrow (S+1)(S-3) = 0$

$\Rightarrow S = -1,\ S = 3$

Remember, we are dealing with age here and therefore $S \neq -1$ because age cannot be negative. So the sister's age is 3.

Substitute $S = 3$ into equation *(3)* to find B:

$B = 3^2 \quad \Rightarrow \quad B = 9 \quad \Rightarrow \quad S = 3,\ B = 9$

Therefore, the sister is 3 and the brother is 9.

15 **E is the answer**

$(2.3) = (2^2 \times 3) + (2-3) \quad \Rightarrow (2.3) = 11$

$(11.4) = (11^2 \times 4) + (11-4) \quad \Rightarrow (2.3).4 = (11.4) = 491$

> ***Exam Tip*** - It is important to be aware of the **common notations** used. The '.' represents a multiplication operation and it may confuse students into thinking 2.3 refers to a decimal rather than the multiply function. An easy way to distinguish this is to look for **consistent usage** of notation within a single question. Since the question $(2.3).4$ has two '.', they have to have the same mathematical meaning, and since we know that the second '.' represents multiply, the first '.' should hence also mean multiply.

16 A is the answer

This question is testing your ability to use **BODMAS** correctly.

Brackets
Other (e.g. indices)
Division
Multiplication
Addition
Subtraction

$$\frac{-((-4)^2-3\times4)^2}{(-3)^3+2\times8} \Rightarrow \frac{-(16-12)^2}{(-27)+(2\times8)} \Rightarrow \frac{-(4)^2}{-27+16} \Rightarrow \frac{-16}{-11} = \frac{16}{11}$$

17 E is the answer

The height, volume and cross section area of the block is related as follows:

$$\text{Height} = \frac{volume}{base\ area}$$

The area of the base is the area of the square with sides $\sqrt{3}-\sqrt{2}\,m$.

$$Base\ area = (\sqrt{3}-\sqrt{2})^2 = (\sqrt{3})^2 - 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2 = 5 - 2\sqrt{6}\,m^2$$

Therefore, the block height can be found as follows:

$$\text{Height} = \frac{volume}{base\ area}$$

$$= \frac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}}$$

$$= \frac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$$

$$= \frac{20\sqrt{2}+8\sqrt{12}-15\sqrt{3}-6\sqrt{18}}{(5)^2-(2\sqrt{6})^2}$$

$$= \frac{20\sqrt{2}+8(\sqrt{4})(\sqrt{3})-15\sqrt{3}-6(\sqrt{9})(\sqrt{2})}{25-4(6)}$$

$$= 20\sqrt{2} + 16\sqrt{3} - 15\sqrt{3} - 18\sqrt{2}$$

$$= 2\sqrt{2} + \sqrt{3}$$

$$= \sqrt{3} - (-2)(\sqrt{2})$$

Hence, $a = 1$, $b = -2$

A is incorrect.
B is incorrect because the coefficients have been mixed up.
C is incorrect because the question asks for the form of $a\sqrt{3} - b\sqrt{2}$.
D is incorrect because the coefficients have been mixed up.

Exam Tip - Remember a common trick to remove surds in the denominator of a fraction is to multiply both the top and bottom of the fraction by the conjugate (complementary) surd. This is the same as multiplying the fraction by 1.

$$\frac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}} = \frac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$$

18 **B is the answer**

Before solving the equation, the value on the right hand side of the equation needs to be manipulated to remove the surd in the denominator. This can be done by multiplying both the top and bottom of the fraction by $4 - \sqrt{3}$:

$$\frac{13}{4+\sqrt{3}} = \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{13(4-\sqrt{3})}{16-3} = 4 - \sqrt{3}$$

Now the equation can be solved by squaring each side:

$$\sqrt{a + b\sqrt{3}} = \frac{13}{4+\sqrt{3}}$$
$$\Rightarrow a + b\sqrt{3} = (4 - \sqrt{3})^2$$
$$\Rightarrow a + b\sqrt{3} = 16 - 2(4)(\sqrt{3}) + 3$$
$$\Rightarrow a + b\sqrt{3} = 19 - 8\sqrt{3}$$

Hence, $a = 19$, $b = -8$

A is incorrect because the coefficients have been mixed up.

C is incorrect.
D is incorrect.
E is incorrect.

19 **B is the answer**

This is a straightforward question which requires the knowledge of how to manipulate surds.

$$x\sqrt{24} = x\sqrt{3} + \sqrt{6} \quad \Rightarrow \quad (\sqrt{24} - \sqrt{3})x = \sqrt{6}$$

$$\Rightarrow x = \frac{\sqrt{6}}{\sqrt{24}-\sqrt{3}} \quad \Rightarrow \quad x = \frac{\sqrt{6}(\sqrt{24}+\sqrt{3})}{24-3} \quad \Rightarrow \quad x = \frac{\sqrt{144}+\sqrt{18}}{21}$$

$$\Rightarrow \quad x = \frac{12+3\sqrt{2}}{21} \quad \Rightarrow \quad x = \frac{4+\sqrt{2}}{7}$$

A is incorrect.
C is incorrect.
D is incorrect.
E is incorrect.

20 **B is the answer**

This question involves manipulating the powers in each equation to form two new simultaneous equations.

$$9^x(27)^y = 1 \qquad (1)$$
$$8^y \div (\sqrt{2})^x = 16\sqrt{2} \qquad (2)$$

Rearrange equation (1) so that each number is represented as a power of 3, collect terms using the rules of indices:

$$9^x(27)^y = 1$$
$$3^{2x} \cdot 3^{3y} = 3^0$$
$$3^{2x+3y} = 3^0$$

Looking at the powers in the above equation, we obtain equation (3):

$$2x + 3y = 0 \qquad (3)$$

Rearrange equation 2 so that each number is represented as a power of 2, collect terms using the rules of indices:

$$8^y \div (\sqrt{2})^x = 16\sqrt{2}$$
$$2^{3y} \div 2^{\frac{x}{2}} = 2^4 \cdot 2^{\frac{1}{2}}$$
$$2^{3y-\frac{x}{2}} = 2^{4\frac{1}{2}}$$

Looking at the powers in the above equation, we obtain equation (4):

$$3y - \tfrac{x}{2} = 4\tfrac{1}{2} \quad \Leftrightarrow \quad 6y - x = 9$$

$$x = 6y - 9 \qquad (4)$$

Substitute equation (4) into equation (3):

$$2(6y-9) + 3y = 0$$
$$12y - 18 + 3y = 0$$
$$15y = 18$$

$$y = 1.2$$

Substitute $y = 1.2$ into equation (4):

$$x = 6(1.2) - 9 \quad \Rightarrow \quad x = -1.8$$

21 **C is the answer**

To answer this question you need to form two simultaneous equations from the information given in the question.

Let Sarah's age be S.
Let Edward's age be E.

'Twice the square of Edward's age subtracted from the square of Sarah's age gives a number equal to 6 times the difference of their ages.'

$$S^2 - 2E^2 = 6(S-E) \qquad (1)$$

The sum of their ages is equal to 5 times the difference of their ages.

$$S + E = 5(S-E) \qquad (2)$$

Next you need to rearrange equation (2) to find E in terms of S:

$$S + E = 5(S-E)$$

$$S + E = 5S - 5E$$
$$6E = 4S$$

$$E = \tfrac{2}{3}S \qquad (3)$$

Substitute equation (3) into equation (1):

$$S^2 - 2(\tfrac{2}{3}S)^2 = 6(S - \tfrac{2}{3}S)$$
$$S^2 - 2(\tfrac{4}{9}S^2) = 2S$$
$$9S^2 - 8S^2 = 18S$$
$$S^2 - 18S = 0$$
$$S(S - 18) = 0$$
$$S = 0 \text{ or } S = 18$$

Since we are looking at age, we must have $S = 18$.

Substitute $S = 18$ into equation (3):

$$E = \tfrac{2}{3}(18) = 12$$

Therefore, we have that Sarah is **18** and Edward is **12**.